## Theory of Lattice Specific heat :-

1) Classical theory ( Dulong -Petit's law)

2) Quantum theory → (a) Einstein theory
→ (b) Debye theory

### (1) Classical Theory : Dulong Petit's law :-

Assumption of this theory :-

According to this theory atoms vibrate like a simple harmonic oscillator & it can have 3 modes of vibration. Atoms are identical classical harmonic oscillators vibrating with same fundamental frequency independent to each other.

i.e. Each atom is equivalent to 3 dim classical H.O.

If there are N atoms in the solid then internal energy of each harmonic oscillator is 3KT.

for a classical H.O. (1 Dim)

$$\text{Total energy} \Rightarrow E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

In 3 Dim, In E there are 6 terms, 2 due to x, 2 due to y, 2 due to z. [3 K.E. term, 3 P.E. term]

& each energy contributing to its half $\left[\frac{1}{2} KT\right]$

i.e. $\frac{1}{2} \times 6(KT) \Rightarrow 3 KT$

→ Internal Energy of classical H.O. is

$$E = 3NK_B T$$

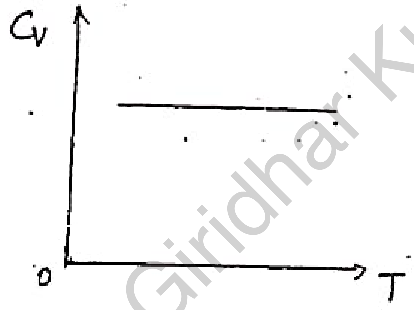
$$\boxed{E = 3RT} \quad \text{i.e.} \quad \boxed{E \propto T}$$

So specific heat,

$$\boxed{C_V = \left(\frac{\partial E}{\partial T}\right) = 3R}$$

Acc to classical theory, specific heat is constant at all temperatures.

But experiments shows that,




This theory fails at low temp.

At low temp,

$$\boxed{C \propto T^3}$$

Hence this theory fails on low temp.

(2) Einstein Theory (Quantum theory) :-

There is only one change, Einstein made in classical theory.

All the assumption of Einstein theory were same as classical theory But the only difference was ; now the Harmonic oscillators are Quantum Harmonic Oscillators.

& They are oscillating with quantised frequencies.

freq. is Quantised in terms of a Q. No. ($n$). & $n$ can take values from $n = 0, 1, 2, 3 \cdots \infty$.

Energy of one-dim Quantum H.O. is

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega} \quad , n = 0, 1, 2, 3 \cdots$$

At temp. t, Average energy of H.O. is

$$\boxed{\bar{E} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{KT}} - 1}}$$

this $\frac{1}{2}\hbar\omega$ is not thermal.

This is the energy of 1 H.O., But Here N, 3D H.O.

N, 3D H.O. is equavelent to 3N, 1 Dim H.O.

So energy of 3N, 1 Dim H.O. is

$$\boxed{\bar{E} = \frac{3N}{2}\hbar\omega + \frac{3N\hbar\omega}{e^{\frac{\hbar\omega}{KT}} - 1}}$$